Determining Surface Tension Using
The Pendant Drop Method

Camille Miller, Jaleel Bolden, Zane Corder, Charlene Higdon
Faculty Advisor: Dr. Guerrero-Millan
Applications
Surface Tension

The energy required to decrease the surface area of a fluid

Surface Tension
Sessile drops

\[ \gamma_{lg} \cos (\theta_C) = \gamma_{sg} - \gamma_{sl} \]
Experimental Setup
Image Processing

Original image

Edge Detected

Edge Plotted

Fitting

Literature: 38.6°  Our code: 38.5°
<table>
<thead>
<tr>
<th>Concentration $V_{\text{Soap}}/V_{\text{Water}}$</th>
<th>Contact Angle</th>
<th>Droplet</th>
</tr>
</thead>
<tbody>
<tr>
<td>5mL/5mL</td>
<td>15.7</td>
<td><img src="image1.png" alt="Image" /></td>
</tr>
<tr>
<td>5mL/10mL</td>
<td>34.7</td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>5mL/20mL</td>
<td>43.5</td>
<td><img src="image3.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Other Methods of Determination

Some instruments to measure surface tension:

- Du Noüy Ring
- Wilhelmy plate
Pendant drop method
Young Laplace Equation

Gravity
\[ \Delta P = \Delta P_0 + (\Delta \rho)gz \]

Capillarity
\[ \Delta P = \gamma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) \]

\[ \gamma \left( \frac{1}{R_1} + \frac{\sin \phi}{x} \right) = \frac{2\gamma}{R_0} + (\Delta \rho)gz \]
Young Laplace Equation

In dimensionless parametric form:

\[ \frac{d\phi}{dS} = 2 - \beta Z - \frac{\sin \phi}{X} \]

\[ \frac{dX}{dS} = \cos \phi \]

\[ \frac{dZ}{dS} = \sin \phi \]

Boundary conditions:

\[ X(0) = Z(0) = \phi(0) = 0 \]

\[ \beta = \frac{(\Delta \rho)gR_0^2}{\gamma} \quad \text{Shape factor} \]

\[ \gamma = \frac{(\Delta \rho)gR_0^2}{\beta} \]
Young Laplace Equation

In dimensionless parametric form:

\[ \frac{d\phi}{dS} = 2 - \beta Z - \frac{\sin \phi}{X} \]

\[ \frac{dX}{dS} = \cos \phi \]

\[ \frac{dZ}{dS} = \sin \phi \]

Boundary conditions:

\[ X(0) = Z(0) = \phi(0) = 0 \]

\[ \beta = \frac{(\Delta \rho)gR_0^2}{\gamma} \quad \text{Shape factor} \]

\[ \gamma = \frac{(\Delta \rho)gR_0^2}{\beta} \]
Young Laplace Equation

In dimensionless parametric form:

\[
\frac{d\phi}{dS} = 2 - \beta Z - \frac{\sin \phi}{X}
\]
\[
\frac{dX}{dS} = \cos \phi
\]
\[
\frac{dZ}{dS} = \sin \phi
\]

Boundary conditions:
\[X(0) = Z(0) = \phi(0) = 0\]

\[\beta = \frac{(\Delta \rho)gR_0^2}{\gamma}\]  \hspace{1cm} \text{Shape factor}

\[\gamma = \frac{(\Delta \rho)gR_0^2}{\beta}\]
Image Enhancement
Edge Detection
Finding $R_0$

$$\gamma = \frac{(\Delta \rho) g R_0^2}{\beta}$$

How did we choose the number of points at the bottom of the drop?
Finding $R_o$
Finding $R_o$

Needle diameter: 0.9 mm

$R_o = 1.5$ mm
Determining the shape factor

**Method 1:**
Two diameters at different heights

\[ \gamma = \frac{(\Delta \rho)gR_0^2}{\beta} \]

**Method 2:**
Fitting the profile
Method 1: Measuring two diameters within the drop ($D_E$ and $D_S$)

$D_E$: Largest diameter in the drop

$D_S$: Diameter $D_E$ to the drop apex

\[ \beta = 0.12836 - 0.7577 \frac{D_S}{D_E} + 1.7713 \left( \frac{D_S}{D_E} \right)^2 - 0.5426 \left( \frac{D_S}{D_E} \right)^3 \]

Method 2: Fitting the whole profile

\[ \beta = 0.05 : 0.01 : 0.5 \]
Method 2: Fitting the profile
# Surface Tension Values $\gamma$ (mN/m)

<table>
<thead>
<tr>
<th></th>
<th>From Literature</th>
<th>First Method ($D_S D_E$)</th>
<th>Second Method (fitting the profile)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water / Air</td>
<td>72.75</td>
<td>73.18</td>
<td>73.47</td>
</tr>
<tr>
<td>Water /10 cSt Silicone oil</td>
<td>-</td>
<td>40.83</td>
<td>40.87</td>
</tr>
<tr>
<td>Glycerol /10 cSt Silicone oil</td>
<td>-</td>
<td>29.19</td>
<td>29.40</td>
</tr>
<tr>
<td>Ethylene glycol /10 cSt Silicone oil</td>
<td>-</td>
<td>18.98</td>
<td>15.74</td>
</tr>
</tbody>
</table>
Measuring the flow rate in electro-coflow
Experimental setup
Drop size measurement
If the same drop appears in several frames...
Monitoring the centroid position
## Results

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average volume of droplets</td>
<td>$1.26 \times 10^{-12} \text{ m}^3$</td>
</tr>
<tr>
<td>Average time between droplets</td>
<td>0.11 s</td>
</tr>
<tr>
<td>Calculated Flow Rate</td>
<td>4.5 microliters per hour</td>
</tr>
<tr>
<td>% Error</td>
<td>4.98%</td>
</tr>
</tbody>
</table>
Thank you!